**2D Hybrid Magnetic Field Model Performance Optimization for Linear Induction Motors**

By

**Michael Thamm**

A Thesis

Submitted to the Faculty of Graduate Studies

through the Department of Electrical Engineering

in Partial Fulfillment of the Requirements for

the Degree of Master of Applied Science

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2021

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**Michael Thamm**

APPROVED BY:

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# ABSTRACT

[Master’s Thesis or Major Research Paper: up to 1 page]

NOTES:

1. Look at troubleshooting documents on desktop and in uwindsor drive as supporting visualizations
2. <https://byjus.com/physics/mesh-analysis/#:~:text=The%20difference%20between%20mesh%20and%20nodal%20analysis%20is%20that%20nodal,used%20for%20calculating%20the%20current>.
3. <https://docs.lib.purdue.edu/open_access_theses/775/>

DEDICATION

[Optional component]

Renee

Areej

Tim

Brad

Solange

# ACKNOWLEDGEMENTS

[Optional component]

Christopher Timperio

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[optional element; if included, list here the name and page number for each table]

# LIST OF FIGURES

[optional element; if included, list here the name and page number for each figure]

# LIST OF APPENDICES

[optional element; if included list here the name and page number for each appendix]

# LIST OF abbreviations/SYMBOLS

|  |  |  |
| --- | --- | --- |
| Category | Symbol | Description |
| Spatial first |  | Number of slots in primary core |
|  | Slot height |
|  | Yoke height |
|  | Tooth width |
|  | Slot width |
|  | Slot pitch |
|  | Air gap |
|  | Aluminum thickness |
|  | Back iron thickness |
|  | Stator length |
|  | Stator height |
|  | Stator depth |
| Electrical Next |  | Phase count |
|  |  | Synchronous velocity |
|  |  | Electrical frequency |
|  |  | Number of magnetic poles |
|  |  | Pole pitch |
|  |  | Peak current |
|  |  | Number of turns per coil |
|  |  | Slots/poles/phase ratio |
|  |  | Conductivity |
|  |  | Relative permeability |
| HM Variables |  | Spatial frequency for nth space harmonic |
| , | Complex harmonic analysis unknowns for nth space harmonic |
|  | Spatial position in the x-direction |
|  | Spatial position in the y-direction |
| MEC Variables |  | Reluctance |
|  | Surface area |
|  | Complex scalar potential |
|  | y-index of a node in the MEC region |
|  | X-index of a node in the MEC region |
|  | Number of rows in a MEC regions |
|  | Number of columns in a MEC region |
|  | Number of nodes in a MEC region |
|  | Number of HM regions in the model |
|  | Number of MEC regions in the model |

|  |  |
| --- | --- |
| Abbreviation | Description |
|  | Two dimensions |
|  | Three dimensions |
|  | Alternating current |
|  | Direct current |
|  | Equivalent circuit model |
|  | Finite element analysis |
|  | Genetic Algorithm |
|  | Hybrid analytical model |
|  | Harmonic model |
|  | Kirchhoff's current law |
|  | Kirchhoff's voltage law |
|  | Magnetic equivalent circuit |
|  |  |
|  |  |
|  |  |

# CHAPTER 1 Introduction

1. Talk about use cases for linear induction motors – maglev Virgin Hyperloop Elon Hyperloop

Linear electric motors [LEM] originated in 1840s with the work of Charles Wheatstone at King's College London,[7]. Since then, many improvements to design efficiency and the manufacturing process have allowed LEMs to be seamlessly integrated into the industries of the modern world. The fundamentals of a LEM are like that of a rotary electric motor [REM] with a significant difference in the force vector produced by the magnetic field interactions. While REMs produce a torque around the secondary (rotor), LEMs produce a force tangential to the winding direction. When choosing between a LEM and a REM for an application it is often ideal to select LEMs when a linear force is required and REMs when a torque is required due to the minimization of lost energy during energy transfer. LEMs are commonly used in precise, high-acceleration applications like actuators and in high-speed, low-acceleration systems like electric trains. The prior classifications are a generalization and with careful design considerations the motor application can satisfy a hybrid of precision, velocity, and force. Additionally, LEMs have an advantage in mechanical simplicity and robustness when compared to REMs due to a lack of moving mechanical parts required to couple the motor to its secondary with a relative velocity.

Now that the use case for LEMs has been defined, the classification of LEMs can be subdivided into further classifications of synchronous and induction designs. Although there are more classifications under the linear AC motor category, these are the major categories in this generation of motor design. The figure below highlights that linear synchronous motors [LSM] are locked to the frequency of the excitation in the multi-phase winding. Alternatively, Induction motors lag behind the synchronous velocity and is actively trying to catch up to this value as measured by the slip of the motor.

Diagram

Description automatically generated

An important similarity between synchronous and induction motor classifications is that they can contain both permanent magnets and winding excitation in either the primary or the secondary of the motor. However, the induction motor can utilize the dense magnetic flux generation that permanent magnets provide in addition to the primary winding excitation. In this paper the focus will shift away from LSM and will be concentrated towards optimization of linear induction motors [LIM]. The majority of equations describing the operation of a LIM are transferable to LSM design but will require further analysis to properly classify and investigate the design of a LSM.

Talk about why LIM is better than LSM for this paper

## *Objectives and Contributions of This Study*

Include a lot of the citations for other papers here

This thesis proposes a holistic program that integrates a genetic algorithm into the hybrid analytical model to optimize a motor within its design space. Due to the flexibility of the hybrid analytical model, with its use of MEC and HM regions, the technique can be implemented for a wide variety of motor classifications. Additionally, by defining the optimization inputs and outputs, any part of a motor model can be optimized, which is a testament to the flexibility of the program.

In this thesis the resulting motor design is optimized for thrust and power to weight ratio by varying the slots and poles of the motor design. The objective is to produce the best-performing motor within a motor classification based on the slot-pole ratio. This is an early optimization in the motor design process and can be coupled with other optimization techniques. The focus on this paper is on constant-length, double-layer single-sided linear induction motors.

## *Organization of Thesis*

Chapter 2 summarizes the fundamental equations regarding electromagnetism. Next, we will discuss the parameters an induction motor requires to produce a relative force. To conclude this chapter, we will discuss the design considerations a designer takes when designing an induction motor.

Chapter 3 summarizes the modelling technique and the optimization algorithm used in this paper and compares them with industry-standard techniques. Computation considerations and efficiency of each are discussed in detail in this chapter.

Chapter 4 summarizes the pre-processed and processed stages of a model solution, where the algorithm structures of each are defined. The pre-processed stage includes defining electrical and mechanical parameters of an induction motor and meshing of the model while the processed stage includes solving the system of linear equations and optimizing the objective functions.

Chapter 5 summarizes the computation considerations and the code side of the model. Includes the results and effectiveness of implementing HAM with GA.

# CHAPTER 2 Induction Motor Theory

## *Converting electrical energy to mechanical energy*

### *Motor Characteristics*

Mmf =ʃ H.dl, while V = Emf = ʃ E.dl

Talk about the diffusion equation and the magnetic vector potential which is approximately related to the B field

I NEED TO FIND AIDAS PAPER SHE SENT ME THAT EXPLAINED THE APPROXIMATION OF THE MAGNETIC VECTOR POTENTIAL AND QUASI STATIC APPROXIMATION I emailed tim and areej the ansys document for plotting in airgap

Any conductor, be it a loop, a coil or simply a piece of plate metal, that is placed in this field will have eddy currents induced in it thus creating an opposing magnetic field, in accordance with Lenz's law.[6] The two opposing fields will repel each other, thus creating motion as the magnetic field sweeps through the metal.

* Convert electrical energy to mechanical energy

The purpose of an electric motor is to transform the electrical energy input into mechanical energy, since they are mutually convertible, by utilizing the principle of induction. When a magnetic field is applied to magnetizable bodies, magnetic flux concentrates throughout the newly magnetized material. This concept is elegantly explained through Ampere’s Law stating that magnetic field strength is strongest when the closed path of magnetic field lines is minimized.

Word

Description automatically generated with medium confidence

Magnetic reluctance is an important material property which quantifies the resistance to a change in magnetic field. The geometry and the permeability of the material directly affect its reluctance. A path of least resistance is produced when the overall reluctance and path length are minimized.

In motor design there are options in the way that the magnetic field is produced which materializes in the form of magnets and current-carrying conductors. Magnets will produce a relatively constant magnetic field that is useful for its magnetization density. Additionally, magnets can operate without reliance on an external source since the magnetic field is produced inherently through the alignment of electron spin throughout the material. Alternately, current carrying conductors produce magnetic field with closed field lines around them. These conductors can be organized into coils and placed in crucial locations throughout a motor to maximize the magnetic field produced.

With the concepts of flux and reluctance defined, magnetomotive force is used to quantify the ability to produce flux. The magnetic circuit model is useful in explaining the behaviour of magnetic field in a material. It is analogous to the electric circuit which defines charge flowing because of a voltage potential through a resistive material. In the magnetic circuit, flux is a consequence of an MMF potential across a source like current carrying conductors in a motor. The flux is resisted through the magnetic reluctance of materials like resistivity in electric circuits. Although the strategic approach of modelling magnetic circuits like electric circuits simplifies the motor modelling, there are important differences.

I need to talk about saturation somewhere since I mention it below

Should I talk about different winding types or the importance of windings and induced flux in the waveform?

[link](https://mospace.umsystem.edu/xmlui/bitstream/handle/10355/4308/research.pdf?sequence=3&isAllowed=y)

* AC Motors vs DC motors
* Linear vs rotary
  + Joule loss by the eddy current in secondary conducting plate
* Multiphase vs single phase windings
  + SWISS s2.2.3

### *Asynchronous multi-phase linear induction motors*

* Motor types that are filtered by linear, AC, asynchronous, multiphase
* Design Parameters of a linear induction motor [link](https://www.linearmotiontips.com/what-are-linear-induction-motors/)
  + Focus on important ones for code, ex) slots, poles, any ratios in geometry
* Performance Parameters and basic equations
  + Christopher Timperio [link](https://www.research-collection.ethz.ch/handle/20.500.11850/379531)
  + Airgap importance
  + Carters Coefficient
* Motor Losses
  + <Although asynch SSLIMs do not experience mechanical do to not needing a gearbox or similar drivetrains>
  + End effects
  + Edge effects
  + Thermal effects
  + Material Loss
  + Search up more
  + Saturation (we can use HAM to check B field based on MEC density)
  + NOTE: I should follow a rule that in this saturation section above I am mentioning it because I can talk about it as a use case of my code. I think I should write a short mention up here that this is important because it will be mentioned later

### *Design Considerations*

* Design Considerations (only things applying to the ratios and python model itself)
  + Geometry limitations that create feasible designs

# CHAPTER 3 Modelling Techniques

## *Industry Standard*

### *Field Modelling*

* Industry Standard (use my powerpoint where I surveyed the industry)
  + MEC vs HM vs ECM vs FEA

The equivalent circuit model [ECM] is the least computationally intensive modelling technique due to a lack of unknown coefficients that require solving in a system of linear equations. Therefore, it does not need a mesh of polygons or boundary conditions. This means that all the variables are solvable prior to implementing the equivalent circuit model which drastically reduces the computation time. The energy transfer between the stator and rotor of a motor is like a transformer equivalent circuit which models the energy transfer from primary to secondary windings. Using Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL), a solution for the circuit is obtained. The solution experiences error introduced with assumptions in the equivalent circuit “components” i.e., resistor, inductor. Additionally, ECM is not a flexible solution for modelling 2D, and 3D geometries because equivalent circuit models cannot be solved as a system of linear equations. Semi-analytical modelling can be used to improve the accuracy of the ECM solution by simulating equivalent circuit components accurately in an FEA model, while introducing computation intensity.

ECM does not account for time and uses average values which adds error

A picture containing text, device, gauge

Description automatically generated

Diagram, schematic

Description automatically generated

Include a picture of a circuit model [link](https://www.electricaldeck.com/2020/11/equivalent-circuit-of-induction-motor.html#:~:text=The%20equivalent%20circuit%20of%20an,characteristics%20of%20the%20induction%20motor.)

While the ECM was very computationally efficient, FEA is the opposite. It is a mesh-based simulation that divides the entire model into a mesh of polygons, where each polygon contains information about the motor’s geometric and magnetic properties. The boundaries between polygons along with the discontinuous border conditions create a system of linear equations that can be solved. The solution is then applied to the mesh to solve the geometric and magnetic properties. This technique is commonly used in a variety of fields of study which require computational modelling. The accuracy of the simulation is proportional to the density of the meshing in the geometry. Logically there is computation time and space complexity which is traded for accuracy due to the size of the computations required for the matrix solution. For this reason, FEA simulations of motors are run at low mesh densities until a final design is ready for characterization. The final solution, simulated with a dense mesh, can be very accurate when compared to a physically built motor.

Magnetic equivalent circuit [MEC] modelling is a subcategory of FEA specific to magnetic circuit modelling. MEC is commonly used in custom modelling methods due to its flexibility and simplicity. Hopkinson's law defines the flux, reluctance, and the MMF which have a superficial resemblance to Ohm’s law when solving electric circuits.

The model works on a conservation of flux principle, stating that the amount of flux entering a polygon must also exit the polygon. Since MEC is a subcategory of FEA it is burdened by large computation requirements and is only useful for niche modelling applications where industry FEA is not applicable.

Harmonic modelling [HM] is a technique used to approximate waveforms with Fourier analysis. The approximation of the waveform is in the form of a summation of N space harmonics, where is the current harmonic number. Each harmonic has its own unknown variables that need to be solved in the matrix equation. The function value at position with a period of defines the function value as a complex Fourier series. Substituting equation X into equation X produces equation X which allows for waveform approximation.

The key computational difference between a mesh-based solution of a geometry and one using HM is how the number of required unknown coefficients scale in size. In the previous section MEC was identified as requiring a dense mesh for accurate computation resulting in a geometric dependance. HM does not rely on the number of polygons in a mesh and instead depends on having enough harmonics to reduce the error in the waveform approximation. However, HM is useful in modelling regions with only homogenous and the other ones like isotropic materials. This means that it is hard to model regions like a motor core due to the variety of materials e.g., copper coils and an iron core. In summary, HM and MEC region accuracy and computation intensity are mutually exclusive. These are important features that will be covered in greater detail in later sections which provide the foundation of the modeling techniques used in this paper.

### *Model/Motor Optimization*

Diagram

Description automatically generated

Image from [link](https://www.researchgate.net/publication/321280564_A_Review_of_Design_Optimization_Methods_for_Electrical_Machines)

NNs consists of a series of algorithms that endeavor to determine and identify patterns

While the modelling methods are important for quantifying motor performance, the model optimization algorithm must also be chosen methodically and implemented effectively. To categorize the field of optimization algorithms, some classifications were provided to simplify the choice. There are 3 main types of optimization algorithms shown in Figure X which serve a similar purpose in the optimization process. The evolutionary algorithm [EA] and the neural network [NN] are both metaheuristics while gradient based algorithms [GBA] require function evaluations to determine search directions. Due to the limitations in complexity and flexibility, GBAs are used for small-scale or local optimizations. Since the model mesh evaluations require metaheuristics, therefore the choice is limited to EAs and NNs. Although NNs are very effective at solving and predicting solutions to complex problems, such as classification, they are computationally intensive. As a result, EAs were chosen as the appropriate optimization algorithm category which includes the genetic algorithm [GA] and particle swarm optimization [PSO], to name a few. These algorithms find a balance between flexibility and efficiency while maintaining a robustness in finding the global maxima and minima. Ultimately, the GA was chosen to optimize the motor model due to its overall effectiveness in solving problems.

[link](https://www.baeldung.com/cs/genetic-algorithms-vs-neural-networks#:~:text=Genetic%20algorithms%20usually%20perform%20well,data%20to%20classify%20a%20network.&text=Genetic%20algorithms%20calculate%20the%20fitness%20function%20repeatedly%20to%20get%20a%20good%20solution)

Compare them in terms of test functions like the Booth function. I can run a test for 3 types of optimizations in C++ to compare the computation time like in Tims paper

## *Hybrid Analytical Modelling Optimization*

Graphical user interface

Description automatically generated with low confidence

### *Hybrid Analytical Modelling*

Previously it was discussed that MEC and HM can be combined to form a hybrid analytical model [HAM]. This model is solved in 2D and is split into orthogonal regions that span the entire periodic length. This is an important feature that distinguishes which regions will become HM or MEC regions. The secondary of the double-layer single-sided linear induction motor [LIM] is infinitely long compared to the finite length of the primary. Longitudinal end-effects are accounted for in this approach which requires air to be modeled to the right and left of the motor core. Similarly, air is modeled above and below the motor to allow for non-continuous boundaries and constrain the model simulation size. Since MEC requires a mesh of polygons, rectangles were chosen to simplify the complexity of the model which must enclose only one type of material. This discretizes the magnetic field produced throughout the model. A model of a double-layer single-sided LIM is shown in figure X below which is segmented into regions. For regions that enclose only homogenous, isotropic, and linear materials, HM will be used to solve for the fields in the motor geometry. Constraining double-layer single-sided LIMs to simple secondary designs makes this a suitable application for HM to model the air around the motor in addition to the secondary of the motor.

MEC regions are designated to sections of the model that are non-homogenous in the longitudinal direction. Since HM does not model these without increasing the computation significantly, MEC regions are assigned to the motor primary to accurately model the complex geometry it encloses. This also provides flexibility in the variety of motor primaries that can be simulated. A common step in the electric motor design process is to carefully choose the tooth geometry to maximize the flux in the airgap while being able to avoid saturation in the motor core. This flexibility also allows complex winding patterns to be simulated within MEC which is a critical optimization in the design process. Assuming that the mesh density can be infinitely dense, any configuration for a primary can be modeled using MEC, which needs to be realized as a feasible design considering computational intensity.

Given the prior classification of HM and MEC regions, regions were solved using HM while region was solved using the MEC. A pre-processed motor model will include many boundary conditions which constrain and define the fields within the model’s domain. Consequently, unknown variables are to be solved within these boundary conditions that are unique to MEC and HM regions. For MEC regions the variable represents the complex potential of its node whereas the variables and represent the unknown coefficients of the nth space harmonic. These variables are discussed in greater detail in later sections of this paper.

## *Genetic Algorithm*

The GA is a kind of evolutionary algorithm that mimics the general concept of evolution. Natural selection is often mentioned in the context of evolution since it is the strong individuals that survive in a given environment. Being the strongest is a generalization that is defined by the objective function of each individual in a population. The structure of a population subject to the GA is visualized in Figure X . The population encapsulates a fixed number of chromosomes, which themselves encapsulate genes. To understand the function of a gene, the application of the algorithm must be defined since the genes are merely input variables to the model that requires solving. The purpose for the GA in this paper is to iterate through motor designs that vary only in their slot-pole combination. Since the slots and poles are inputs to the model, they are considered the genes through the nomenclature of the GA.

Diagram

Description automatically generated

### *Crossover*

Like real life, the GA has core functions that are appropriately named after events in the natural process of evolution. Crossover is one of these functions. It allows parent chromosomes to exchange their genes and produce child chromosomes while retaining some of their original genes within the chromosome. The ratio of genes that will be overwritten is defined by a crossover point as visualized in Figure X. Although this diagram is useful in explaining the concept of crossover, it does not represent the true number of genes per chromosome that are used in this paper. Additionally, the values of the genes were limited to binary values for simplicity, but the true values can contain any format such as integers for slots and poles. Since the crossover point acts determines the percentage of genes shared among parent chromosome, it is important to not choose too small or large of a ratio due to solver robustness. If a small percentage of genes in the chromosomes were crossed over then the solver may become stuck in local minima or maxima rather than the desired global alternative. Alternatively, a large percentage of genes crossed over between chromosomes will have large variations in the solution and can cause an instability in the solver.

Diagram

Description automatically generated with low confidence

### *Mutation*

Mutation is another important function of the GA and is responsible for manipulating the values of randomly selected genes within a chromosome. The probability that this function occurs shall remain low to maintain solver robustness. There are many ways that the mutation function can manipulate the values of the genes and they . The general concept of mutation is visualized in Figure X , which highlights the genes that were randomly selected for mutation.

Diagram

Description automatically generated

### *Selection*

Selection is a function that identifies the strongest chromosomes among the population. This identification process is achieved with a fitness function which is application specific. For this paper the fitness function measures the performance metrics of the motor after having solved the mesh. The performance metric values are then either maximized, minimized, or centered around a bias. After the solver has produced many generations of the population, the resulting slot-pole ratio will produce a motor with fine-tuned performance. Some common performance parameters of a motor are shown in Figure X. Although there are more performance parameters that indicate other useful characteristics of the motor, the GA solver is not efficient in large multi-objective optimizations.

Diagram

Description automatically generated

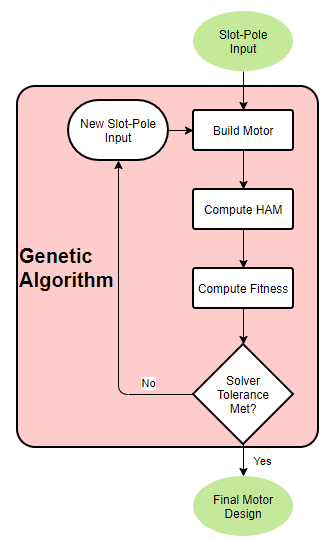
Optimizing for thrust, mass, and efficiency tunes this application to produce competitive motors. Since they are all relatively important performance parameters it is important that the solver produces pareto-optimal solutions, meaning the solution equally satisfies the fitness function criteria. A solution that is not pareto-optimal will still optimize every multi-objective variable but with an inequal emphasis. This information allows the solver to select the strongest chromosomes among the group which will then be subject to crossover to exchange their advancement towards solver convergence with their child chromosomes. Since the population size must remain constant, the weakest chromosomes are then removed from the population and discarded. This ensures that the solver does not deviate from convergence. To achieve solver convergence, the current generation must have only small variations during crossover to produce children chromosomes. In other words, the solver has converged when every subsequent generation produces a very similar motor design.

[link](https://towardsdatascience.com/introduction-to-genetic-algorithms-including-example-code-e396e98d8bf3)

The GA is the base of NSGA-II which is a multi-objective wrapper around GA that has non-dominated solutions [Link](https://www.sciencedirect.com/science/article/pii/S1877705811022466#:~:text=NSGA%2DII%20is%20a%20well,or%20maximizing%20the%20machining%20performances.)

# CHAPTER 4 Model Structure

Due to the size and complexity required to build a HAM it is important to simplify the model into smaller procedures. The image below highlights the state transitions made by the model to produce a pre-processed motor, solve the motor, and produce a processed motor model. The motor performance parameters are then used to compute the genetic algorithm objective function value and compare it to a desired solver tolerance. Prior to building a pre-processed motor, the genetic algorithm will provide a weighted input in the form of slots and poles in the primary of the motor. Converging towards solver tolerance with the objective function indicates that the resulting performance parameters are optimized for a given slot-pole ratio of a motor.

**

I think it is important that this stays here above the build motor and grid section

## *Pre-Process*

### *Build Motor and Grid*

* All the characteristic equations used to build a motor

There are many important relationships between the motor parameters which can be taken advantage of to assign ratios between variables. Creating relationships between variables allows for more flexibility in the model if the range of inputs produce feasible motor designs. This is an important step to constrain the complexity of the optimization space. One important relationship is between the slot and tooth width of the primary core. Since saturation degrades the motor performance in teeth that do not provide enough volume for the flux, the tooth width must not be too small in a motor design. Alternatively, the slot width should not be made too small producing unrealized potential. A relationship where the tooth width is roughly three eighths of the slot pitch produces a slot width to tooth width ratio that will work for a large range of motors that vary in their slot and pole pitch. The ratio can also be removed to simulate the motor with mutually exclusive slot and tooth widths.

In equation lambda there is a relationship to the stator core length and slots of a stator. Since the number of slots is an input received from the GA, slot pitch varies with each motor design iteration. The purpose of the optimization in this paper is to test fixed length motor designs, while varying only the slot-pole ratio of the motor design iterations. This means that motor optimization for different stator core lengths require the simulation to be repeated unless the stator core length is passed as an input variable to the GA.

self.f = self.vel/(2\*self.Tp)

is a good relationship for frequency

MAYBE include a table or mention a table of all the values used to build the motor and credit 2019 paper

Before a motor model can be solved with the HAM it must first have a mesh for the motor geometry. A rectangular mesh is produced by discretizing the model and prioritizing the motor core geometry. Since the slot and coil geometries are generally the most complex, the mesh density in the x and y direction is proportional to the complexity of the core shape. The MEC and the HM regions are unique in their solution methods so the index variables will be unique as well. For the MEC region, variable will be used for the node index in the x-direction while is the node index in the y-direction. The finite index limit for these two index vectors are defined as and , which can be multiplied to produce the total number of unique indexes . It was previously mentioned that MEC modeling requires the boundaries of each node to enclose only one material type. This concept along with the regions can be seen in Figure X below.

Graphical user interface

Description automatically generated

The lengths of a node in the x and y direction are and respectively which defines the dimensions of the rectangle nodes throughout the mesh i.e., the value of in the MEC region is constant throughout all other regions along the -direction with a constant and vice versa. The left and right boundary coordinates in the x-direction are assigned to and respectively. To maintain periodicity in the x-direction, the nodes on the x-boundaries where = 1 and = L are coupled. This property states that:

| ***current* node x index** | ***left* node x index** | ***right* node x index** |
| --- | --- | --- |
| = 1 |  |  |
| = |  |  |
| 1 < < |  |  |

MAYBE INCLUDE A PICTURE OR DIAGRAM WITH A MESH FOR COUPLING.

Chart, box and whisker chart

Description automatically generated

Now that the size and density of the mesh has been defined, it is important to define the properties of each individual node within the mesh. Each node has a reluctance, flux, and MMF component which is defined by the material the node encloses. The relative permeability (, the vacuum permeability (, and the cross-sectional area ( are all required to define the reluctance of a node:

Since the material in the node is homogenous, the reluctance near the positive boundary is equal to the reluctance near the negative boundary for x and y directions. The flux contained in each node at a given time is written as:

The conservation of flux is maintained in equation X stating that all flux entering one potential node should be equal to the magnetic flux leaving the node. Variables , , , and are the indices of the neighbouring nodes. The source term producing the flux is the MMF generated by the coil. The term is a variable for the MMF contained in the node. The MMF in a coil is calculated based on the current excitation (, the number of turns in a coil (), the number of nodes in the x-direction for a single coil (, and the y-position of the node in the coil (.

SHOW THE EQUATIONS FOR CURRENT HERE and explain them

The y-position of the node in the coil helps to determine how long the flux path will be. Nodes that are positioned near the bottom of the teeth will enclose the whole area of the coil and produce a longer flux path, leading to an increase in .

SHOW THE diagram of the scaling (should make my own)

“The magnitude was maximal in the yoke elements, as the formed magnetic path through the air gap enclosed the whole area of the coil”

Due to the merging of MEC and HM, the unknown variable for the potential of the node arises in the flux equations above. This value is calculated in equation X:

The vector potential can be broken down into its time and space dependent parts. The time dependent complex exponential is defined by the frequency (), the time (, and the complex notation . Alternatively, the space dependent part is an unknown variable that requires a system of linear equations to solve which is discussed further in a later section.

To quantify the HM regions, the equations for magnetic flux density and magnetic field strength materialize in the form of a complex Fourier series. The equation parameters change from in the MEC equations to since the HM does not require discretized points and is solvable for any coordinate in the model. Since the MEC model determines the mesh density, the HM model follows suite and will be calculated at the center of a node for a processed solution. The vector potential was mentioned earlier in this paper through the diffusion equation X.

The vector potential is defined as a complex Fourier series in the form of:

The relationship between the magnetic flux density and the vector potential was defined in equation X and allows for the solution of the tangential and normal component of the magnetic flux density equations:

Where:

These equations produce the solution of the magnetic flux density for one periodical length and N spatial harmonics, where one space harmonic is defined as . Since the HM region contains the same material throughout the region, the values , , and are independent of a node index within the region. The relative velocity between the primary and secondary is defined as . The unknown variables and are like of the node in the MEC. The same system of linear equations is used to solve these unknowns which requires boundary condition equations to define.

### *Compute HAM*

Now that the required mesh parameters have been defined, the construction of the system of linear equations relating the unknown variables can begin. The boundary condition between two neighbouring regions can be between two HM regions, between MEC and HM regions, or it can be non-continuous. This classification defines which unknown variables, defined by regions surrounding the boundary, are included in the equation. Since sources cannot be infinite in magnitude and the air surrounding the model theoretically extends to infinity, the Dirichlet boundary condition applies. The 2018 paper defines this as forcing all the field components to vanish at the boundary. This equation applies to regions , and is defined as:

For continuous boundaries, the normal and tangential components of each neighbouring region must be conserved. This is true for HM-HM boundaries as well as HM-MEC boundaries. Where is the lower region index at the boundary positioned at .

Where:

The HM-MEC boundary must be expanded upon equation X to couple the Fourier and MEC solutions. Unlike the MEC region, the HM regions do not produce a source. This means that the transfer of energy into the HM region is conserved at the HM-MEC boundary. Equation X(By=By) can be implemented at the boundary using Equation X (flux conservation one) to produce the equations:

The flux in the normal direction in equation X is then substituted with equation X which explains that the magnetic flux is equal to the average flux density times the cross-sectional area at the boundary. The equation for the flux with the depth of the domain is defined as:

The solution for was solved using the same steps as in equations X and Y above by substituting with instead of . While the equations above define the normal boundary condition, the equations below define the tangential boundary condition. For this boundary, equation X(Hx=Hx) can be expanded as:

Both sides of the equation are in the form of a complex Fourier series as seen by the summation across harmonics. Discretizing the coils into nodes of a mesh creates a staircase shaped waveform which indicates that the Fourier series needs to be modified for a piece-wise continuous function value. This concept is shown in equation X which expands on the Fourier equations X, Y, Z. The value for is substituted with a summation of nodes in the x-direction for .

Some of the variables that help solve for the function value depend on the position of the node at index (. The tangential magnetic flux density of a node is equal to the average flux in the x-direction divided by the cross-sectional area of the flux direction (x-direction):

To produce a processed mesh model, the equations for each boundary condition are separated into a matrix of coefficients , a matrix of unknown variables , and a matrix of constants . Table x below expands on the matrix equation :

|  |  |  |  |
| --- | --- | --- | --- |
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|  | . |  |
| . | . | . |
| . | . | . |
| . | . | . |
|  |  |  |

Where the dimensions of the square matrix are , where is the total number of HM regions in the model and is the number of MEC regions in the model. M is defined as the number of nodes in the MEC region and N is defined as the number of harmonics in the waveform approximation. The dimensions of the column vectors and are . To optimize the system of linear equations, the equations and coefficients that are solvable in the pre-processing stage can be removed. In the Dirichlet equations, an infinite position drives the unknown coefficients to and 0 for and respectively.

|  |  |  |
| --- | --- | --- |
|  |  |  |

Alternatively,

|  |  |  |
| --- | --- | --- |
|  |  |  |

These equations can now be removed from the equation set along with the and unknown variables. The removal of 2N equations and 2N unknown variables maintains a square matrix A which has the new dimensions of . The system of linear equations is then solved using lower-upper-decomposition to produce the unknown variables of the HM and MEC regions.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Type | Continuous | | MEC | Non-continuous |
| Top | Bottom |
|  |  |  |  |  |
|  |  |  |  |  |

## *Post-Process*

### *Update Grid*

Now that the unknown variables for the HM and MEC regions are solved, their values can be substituted into the model equations to solve for any processed mesh parameters. An important performance parameter used in the GA objective function is the thrust of the motor. The force on the primary of the motor has a normal and tangential component which can be calculated with the equations below:

Show the thrust equations and the thermal equations

These equations were derived from the Maxwell stress tensor in the airgap where the complex conjugate of a complex variable is denoted with a \* in the superscript.

Talk about the other objective function solutions only if they are post processed

The conduction loss in the secondary of the motor can be calculated using the Poynting vector, applied in the air gap.

With this information a rough estimate on the efficiency of the motor can be produced which doubles as a objective function value for the GA.

### *LIM\_Show*

A specific field type can be plotted on the mesh to visualize its magnitude throughout the mesh. The magnitude for the range of values for the field type are translated to a colour gradient and plotted on the mesh to define the field. This is important for visualizing the magnetic flux density in the core of the primary to check that the core dimensions are compatible with the localizations of flux density throughout the core.

The tangential and normal magnetic flux densities in the middle of the air gap, between the primary and secondary, can be plotted to check the piece-wise continuous waveform produced by the coils in the primary.

### *Platypus*

* Python function for checking model integrity
* Talk about the objective function multivariables each in detail. Ex thrust is generated throgh flux linkage in airgap
* (provide the link of the module?)

# CHAPTER 5 Proof of Results

## *Compare to FEA*

### *Sub-Section Heading Here*

* Compare to FEA
  + Multiple sims with results of
    - B field in core
    - B field in airgap
    - Thrust plots
  + Magnetostatic vs Transient
* Error
  + Quantization
  + Lu decomp
  + Fourier harmonic number (approximation of the waveform)
  + MEC discretization error
  + Assumptions made in math in the 2019 paper (top of page)

## *Convergance of GA towards improvement*

### *Multi-Objective Function % Improvement*

### *non-dominated or pareto optimal solutions*

* Convergance to improvement for GA

# CHAPTER 5 Computation Considerations

## *Section Heading Here*

### *Sub-Section Heading Here*

* MEC and HM scale in computation time through mesh and harmonics
* Efficiency
  + Memory
  + Time Complexity
* Hyperparameter Optimization
* Computer Hardware Considerations
* Multiprocess/Coroutine
* Caching
* Class Inheritance

# REFERENCES/BIBLIOGRAPHY

*[Ensure all citations are formatted in the same style. The style you choose is based on your departmental/discipline standard – refer to your advisor/thesis committee for guidance]*

# APPENDICES

## Appendix A

*[If applicable, include copyright permission for previously published material. Remove any personal information from appendices and forms, such as emails, phone numbers, signatures, etc.]*

# VITA AUCTORIS

*[The Vita Auctoris is a required thesis element, however, there are no specific requirements / restrictions about its format or contents. It should include as a minimum the author's name, education and degrees. A sample approval page is provided below. For more details refer to the* [*formatting requirements webpage.*](https://www.uwindsor.ca/graduate-studies/385/format-requirements#vita) *]*

|  |  |
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